Dynamic Tax Incidence with Heterogeneous Households

John H. Boyd III

Revised: October 19, 1999

Abstract. This paper examines the utility gains and losses induced by changes in capital taxation in an economy with heterogeneous discount factors. A Ramsey equilibrium, where households earn wage income and accumulate capital, but may not borrow against future wage income, provides a natural setting for this analysis. In the short run, the agents with little or no capital gain from increased transfers following an increase in taxation. In the long run, everyone loses as the capital stock declines. The households with little or no capital are poor because they are relatively impatient. As a result, they prefer to get the short-run gains from taxation, in spite of the long-run (and thus heavily discounted) losses.

Department of Economics, Florida International University, Miami, FL 33199

Typeset using $\mathcal{AMS}$-TEX

1. Introduction

Do all households prefer a cut in capital income taxes? Not if they have different discount factors. This paper examines the utility gains and losses induced by changes in capital taxation in a heterogeneous household economy. In contrast with the representative agent case, there can be both winners and losers from changes in taxation. In the short run, the agents with little or no capital gain from increased transfers following an increase in taxation. In the long run, everyone loses as the capital stock declines. The households with little or no capital are not poor by accident. They are poor because they are relatively impatient. As a result, they prefer to get the short-run gains from taxation, in spite of the long-run (and thus heavily discounted) losses.

A Ramsey equilibrium provides a natural setting for this analysis. Households earn wage income and may accumulate capital, but are not permitted to borrow against future wage income. This insures that all households have positive consumption in the steady state. Capital is taxed, and the proceeds are distributed equally to all households. The economy starts at the steady state. Tax rates are then suddenly and unexpectedly changed.

When gross capital income is increasing in capital stock, a turnpike theorem is available for the Ramsey equilibrium. For small changes in the tax rates, it can be used to analyze the effects of small increases in capital taxation for a variety of preferences and technologies.

In the special case of logarithmic preferences and a Cobb-Douglas technology, the transition path to the new steady state is explicitly calculated. Further, the utility gain or loss for each agent is also calculated. These agents have single-peaked preferences over tax rates. The most patient household is the only one preferring no tax. All of the other households prefer some tax, even though their steady-state consumption would be higher without a tax.

A number of economists have used intertemporal models to look at capital income taxation. The first pioneering efforts of Feldstein (1974a, b) used exogenously given savings functions that differ for capital and labor income. Feldstein's most striking result was that capital income taxes could be shifted to labor in the long run through a reduction in the capital stock. This result has been typical of the models inspired by Feldstein's work. One interesting variant was pursued by Homma (1981) who performed a similar analysis for the Pasinetti model. This enabled him to investigate the effects on income distribution.

More recent work has focused on the factors responsible for the shifting, and the welfare effects of the tax. Wildasin (1984) focuses on the crucial role played by the government's propensity to save. Boadway (1979) notes that the shifting only occurs in the long run. He argues that it could easily take 60 or more years before labor feels the shifting. He goes on to conjecture that workers could be better off in spite of their long-run loss if they are relatively impatient. Both Becker (1985) and Chamley (1981) use equilibrium models based on a representative consumer. This makes a utility-based welfare analysis possible.

The Ramsey equilibrium shares many points in common with Pasinetti's two class model (1962). The Pasinetti model has two classes of households, capitalists and workers. Each has an exogenously given savings rate. As Wan (1971) notes, the steady-state values of all of the important economic variables are determined by the capitalist's savings function. These include the capital stock, the rate of return on capital, the wage rate, the output per worker, and the capital-output ratio. This is equally true of the Ramsey equilibrium with the discount rate of the most patient household playing the role the capitalist's savings rate does in the Pasinetti model.

One way to capture this two-class structure in an optimizing model is to bar saving and lending by workers. A comprehensive analysis of this type of model may be found in Judd (1985). An alternative is to employ the Ramsey equilibrium (Becker, 1980; Becker and Foias, 1987; Becker, Boyd and Foias, 1991). This permits the combination of features from all of the above analyses. The distinction between capitalists and workers arises endogenously. The most patient household (the capitalist) ends up with all of the capital in the steady state. The other households (the workers) hold none. If a labor-leisure choice is included at each point in time, the capitalist will perform no labor, provided that there are enough workers around to support him in accustomed style. When taxes are changed unexpectedly, the whole dynamic transition path may be analyzed. In contrast to Feldstein, the model is designed to perform a balanced-budget analysis. The tax proceeds are redistributed equally to all. The government's savings propensity does not enter at all. Any such effects can only operate through its ghost, the savings propensities of those who receive the tax money.

The Ramsey equilibrium with taxation is developed in section two. Section three describes the turnpike property enjoyed by the Ramsey equilibrium with taxation, and characterizes the steady state. Section four focuses on the case of gross capital income taxation, and examines the effects of an unexpected permanent increase in capital taxation. Section five contains an example where the transition path can be calculated. The extra information here allows me to find a politico-economic equilibrium, where no alternative tax rate will win a majority rule election. Concluding remarks are in section six.

2. The Ramsey Equilibrium

Throughout the paper, I maintain the following assumptions, most of which are fairly standard. There are H households, each having additively separable utility. Household h's felicity function $u_h \colon \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable on $\mathbb{R}_{++}$ with $u_h' > 0$, $u_h'' < 0$ and $u_h'(0+) = \infty$. Only consumption affects felicity. Household h discounts future felicity at rate δ_h. Households are labelled so the discount factors obey $1 > \delta_1 > \delta_2 >$

$\cdots > \delta_H > 0$. Household h has labor endowment $L_h > 0$, and the total labor endowment of the economy is $L_0 = \sum_{h=1}^{H} L_h$. Let $\ell_h = L_h/L_0$ denote the labor share of household h. I will sometimes refer to household one as the "capitalist" and the other households as the "workers". In fact, if given a labor-leisure choice, the capitalist will often choose to not work. The workers would always choose to work.

The technology is described by a constant returns to scale production function $F(K, L)$. With competitive firms, all labor will be employed. We focus on a production function written only in terms of capital. Define $f(K) = F(K, L_0)$. Assume $f \colon \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable on $\mathbb{R}_{++}$ with $f(0) = 0$, $f' > 0$, $f'' < 0$, $f'(0+) = \infty$ and $f'(\infty) = 0$. In addition, I assume that gross capital income $Kf'(K)$ is increasing in capital stock K. This last condition permits me to employ the turnpike results of Becker and Foias (1987, 1990, 1994).

In this model, capital earns a return q_t, which is taxed at rate $\tau(q_t)$, $0 \le \tau(q) < q$ for $q \ge 1$. We will assume $1 > \tau' \ge 0$. Two taxation possibilities that fit into this framework are a tax at rate ϑ on the gross return to capital ($\tau(q) = \vartheta q$), and a tax at rate ϑ on the interest earned by capital ($\tau(q) = \vartheta(q-1)$). The proceeds of the tax are transferred back to the households. Total transfers at time t are T_t. Household h receives a share $0 < \theta_h < 1$ of total transfers. The ratio $\xi_h = \ell_h/\theta_h$ of wage-share to transfer-share is important in the following analysis. Wages and gross return to capital at time t are w_t and q_t, respectively. Household h chooses consumption c_t^h and capital holding x_t^h to solve

$$C^h(w_t, q_t, \tau, T_t, x^h) = \max \sum_{t=1}^{\infty} \delta_h^{t-1} u_h(c_t^h)$$

$$\text{s.t. } c_t^h + x_t^h = L_h w_t + (q_t - \tau(q_t))x_{t-1}^h + \theta_h T_t, t \ge 1$$

$$c_t^h, x_t^h \ge 0;\ x_0^h = x^h.$$

The production sector maximizes profits and must solve a problem of the form

$$P(q,w) = \max\{[F(K,L) - qK - wL] : K, L \geq 0\}$$

in every time period.

RAMSEY EQUILIBRIUM. A sequence $\{q_t, w_t, K_{t-1}, c_t^h, x_{t-1}^h, T_t, x^h\}_{t=1}^{\infty}$ is a Ramsey equilibrium with capital taxation if:

A) c_t^h and x_t^h solve $C^h(w_t, q_t, \tau, T_t, x^h)$ for all h

B) (K_{t-1}, L_0) solves $P(q_t, w_t)$

C) $\sum_{h=1}^{H} x_{t-1}^h = K_{t-1}$

D) $\tau(q_t)K_{t-1} = T_t$.

Thus, consumers maximize utility (A) and producers maximize profits (B). Note that the labor market clearing condition has been incorporated into (B). This is possible due to the constant returns to scale. These imply profits are zero and wages are given by $w_t = F_L(K_{t-1}, L_0) = [f(K_{t-1}) - q_t K_{t-1}]/L_0$. Capital markets clear (C) and the government obeys its budget constraint (D).

3. Convergence of Ramsey Equilibrium

The theory of the Ramsey equilibrium has been developed by Becker (1980), Becker and Foias (1987, 1990, 1994), Becker, Boyd, and Foias (1991), and Sorger (1994). Many of the basic results carry over to the Ramsey equilibrium with taxation.

It is straightforward to extend the equilibrium existence arguments of Becker, Boyd, and Foias (1991) to accommodate taxation.

3.1. Steady States

The steady state was first described by Becker (1980). In his steady state, the most patient household owns all the capital, and the other households consume only their wage income. An examination of the first-order conditions for the households and firms shows that Becker's description applies here also, with appropriate modifications to include taxes and transfers.

THEOREM 1. *Steady-state Ramsey equilibria have the form* $\{\bar{q}, \bar{w}, \bar{K}, \bar{c}^h, \bar{x}^h, \bar{T}, \bar{x}^h\}$ *obey the following:*

(1) $\bar{q} = f'(\bar{K})$.

(2) $\bar{w} = [f(\bar{K}) - \bar{q}\bar{K}]/L_0 = [f(\bar{K}) - \bar{K}f'(\bar{K})]/L_0$.

(3) $\bar{T} = \tau(\bar{q})\bar{K}$

(4) $\bar{x}^1 = \bar{K}$ *and* $\bar{x}^h = 0$ *for* $h \neq 1$.

(5) $\bar{c}^1 = \ell_1\bar{w} + (\bar{q} - \tau(\bar{q}) - 1)\bar{K} + \theta_1\bar{T} = \ell_1[f(\bar{K}) - \bar{K}f'(\bar{K})] + [f'(\bar{K}) - 1 - (1 - \theta_h)\tau(f'(\bar{K}))]\bar{K}$.

(6) $\bar{c}^h = \ell_h\bar{w} + \theta_h\bar{T} = \ell_h[f(\bar{K}) - \bar{K}f'(\bar{K})] + \theta_h\tau(f'(\bar{K}))]\bar{K}$ *for* $h \neq 1$.

3.2. The Equilibrium Path

The basic characterization of the equilibrium path by Becker and Foias (1987, 1990, 1994) also carries over. They first show that $x_t^h = 0$ infinitely often along optimal paths. Their arguments are based on the first-order conditions for the consumer's problem. To show the flavor, I reprove one of their key lemmas for the case with taxation.

LEMMA 1. *If* $\{q_t, w_t, K_{t-1}, c_t^h, x_{t-1}^h, T_t, x^h\}_{t=1}^{\infty}$ *is a Ramsey equilibrium, then* $\liminf_{t\to\infty} \delta_1[q_t - \tau(q_t)] \leq 1$.

PROOF. Apply the first-order condition for consumer 1, $\delta_1[q_{t+1} - \tau(q_{t+1})]u_1'(c_{t+1}^1) \leq u_1'(c_t^1)$ to find:

$$\prod_{t=1}^{T} \delta_1[q_{t+1} - \tau(q_{t+1})] \leq \frac{u_1'(c_1^1)}{u_1'(c_{T+1}^1)} \leq \frac{u_1'(c_1^1)}{u_1'(K^M)}$$

where k^M is the maximum of the maximum sustainable stock and the initial aggregate capital stock $\sum_{h=1}^{H} x^h$. As $c_1^1 > 0$ due to the Inada condition on u_1,

$$\limsup_{T\to\infty} \prod_{t=1}^{T} \delta_1[q_{t+1} - \tau(q_{t+1})] < \infty,$$

from which it follows that $\liminf_{t\to\infty} \delta_1[q_t - \tau(q_t)] \leq 1$. $\square$

Summing up the characterization from Becker and Foias (1987), we have:

THEOREM 2. *If* $\{q_t, w_t, K_{t-1}, c_t^h, x_{t-1}^h, T_t, x^h\}_{t=1}^{\infty}$ *is a Ramsey equilibrium, then there exists a time* T *such that* $x_t^h = 0$ *for* $t > T$ *and* $h \neq 1$. *Moreover* K_t *converges to* $\bar{K}$ *with* $\delta_1[f'(\bar{K}) - \tau(f'(\bar{K})] = 1$.

Once the equilibrium has reached the state where all of the capital is owned by household one, convergence to the steady state is monotonic. Becker and Foias (1990, 1994) show monotonicity under the hypothesis that the patient household's equilibrium income is increasing in K. That income, $g(K)$, is given by

$$\begin{aligned} g(K) &= \ell_1\bar{w} + (\bar{q} - \tau(\bar{q}))\bar{K} + \theta_1\bar{T} \\ &= \ell_1[f(K) - Kf'(K)] + [f'(K) - (1-\theta_1)\tau(f'(K))]K. \end{aligned}$$

THEOREM 3. *Suppose* $g' > 0$. *If* $\{q_t, w_t, K_{t-1}, c_t^h, x_{t-1}^h, T_t, x^h\}_{t=1}^{\infty}$ *is a Ramsey equilibrium with* $x_t^h = 0$ *then* K_{t-1} *converges monotonically to the steady state.*

Now $g' = -\ell_1 Kf'' + [Kf'(K)]' - (1-\theta_1)[\tau(f'(K))K]'$. Taxing either the gross return to capital or merely the interest insures $g' > 0$. When $\tau(q) = \vartheta q$,

$$g' = -\ell_1 Kf'' + [1 - \vartheta(1-\theta_1)][Kf']' > 0,$$

and when $\tau(q) = \vartheta(q-1)$,

$$g' = -\ell_1 Kf'' + [1 - \vartheta(1-\theta_1)][Kf']' + (1-\theta_1)\vartheta > 0.$$

4. Unexpected Changes in Tax Rates

We now specialize to the case where $\tau(q) = \vartheta q$ with $0 < \vartheta < 1$ and examine changes in the tax rate. The economy starts in a steady state. The most patient household (the capitalist) owns all the capital, the rest (the workers) receive only wage income. The tax rate is suddenly and unexpectedly permanently changed. The current state is no longer the steady state under the new tax regime. The analysis of tax change proceeds in two parts. First, I analyze the steady-state effects of the change. Second, I combine a turnpike theorem with the steady-state results to investigate the effects on the transition path. This second portion of the analysis is the more important. It yields a welfare analysis of the tax changes.

4.1. The Steady Effects of Taxes

Let $c^h(\vartheta)$, $x^h(\vartheta)$, $K(\vartheta)$, $q(\vartheta)$, $w(\vartheta)$ and $T(\vartheta)$ denote the steady-state consumption and capital holdings of household h, aggregate capital stock, (gross) return to capital, wages and transfers, respectively. Using Theorem 1, we find $\delta_1(1-\vartheta)q = 1$, and that $x_1(\vartheta) = K(\vartheta)$ obeys $\delta_1(1-\vartheta)f'(K(\vartheta)) = 1$ since $q(\vartheta) = f'(K(\vartheta))$. Plugging into the expression for wages shows $w(\vartheta) = [f(K(\vartheta)) - K(\vartheta)f'(K(\vartheta))]/L_0$. Similarly $T(\vartheta) = \vartheta q(\vartheta)K(\vartheta) = \vartheta K(\vartheta)/[\delta_1(1-\vartheta)]$.

Further, consumption by household h is $c^h(\vartheta) = L_h w(\vartheta) + \theta_h T(\vartheta)$ for $h > 1$ and $c^1(\vartheta) = L_1 w(\vartheta) + [(1-\vartheta)q(\vartheta) - 1]K(\vartheta) + \theta_h T(\vartheta)$. Let $\varepsilon = f'/Kf''$ denote the elasticity of demand for capital.

Grinding through the derivatives shows the effect of changes in the tax rate on steady state values of all variables:

$$\frac{dq}{d\vartheta} = \frac{q}{1-\vartheta} = \frac{f'}{1-\vartheta} = \frac{1}{\delta_1(1-\vartheta)^2} \tag{3.1}$$

$$\frac{dK}{d\vartheta} = \frac{1}{f''}\frac{dq}{d\vartheta} = \frac{f'}{(1-\vartheta)f''} = \frac{1}{\delta_1(1-\vartheta)^2 f''} \tag{3.2}$$

$$\frac{dw}{d\vartheta} = -\frac{Kf''}{L_0}\frac{dK}{d\vartheta} = -\frac{K}{L_0\delta_1(1-\vartheta)^2} \tag{3.3}$$

$$\frac{dT}{d\vartheta} = (Kf'' + \vartheta f')\frac{dK}{d\vartheta} = Kf''(1+\vartheta\varepsilon)\frac{dK}{d\vartheta} \tag{3.4}$$

$$\frac{dc^h}{d\vartheta} = \theta_h\frac{dT}{d\vartheta} + \ell_h\frac{dw}{d\vartheta} = \frac{\theta_h Kf'(1+\vartheta\varepsilon-\xi_h)}{1-\vartheta} \qquad \text{for } h > 1 \tag{3.5}$$

$$\frac{dc^1}{d\vartheta} = \frac{\theta_1 Kf'(1+\vartheta\varepsilon-\xi_1)}{1-\vartheta} + (\delta_1^{-1}-1)\frac{dK}{d\vartheta} \tag{3.6}$$

This yields the following effects of a tax increase on the steady state: The gross return to capital increases (3.1). Capital stock decreases (3.2). Wages decrease (3.3). Transfers increase if $\vartheta Kf'' + \vartheta f'$ is positive and decrease if it is negative (3.4). The assumption that $[Kf']' > 0$ implies transfers increase. Consumption by the workers ($h > 1$) increases if $1 + \vartheta\varepsilon - \xi_h$ is positive and decreases if it is negative (3.5). (When $\xi_h = 1$, consumption decreases.) Consumption by the capitalist (3.6) is less than consumption by a worker with the same labor/transfer ratio ξ.

4.2. The Transition Path

As in Feldstein's models, the burden of the tax, in terms of factor prices, is shifted to the workers in the long run. We should not leap to the conclusion that the workers are worse off. As Boadway (1979) has pointed out, it can take a long time to get to the long run, and the workers can be better off in the short run. In return for the long-run loss, there are short-run gains by the workers due to the large initial transfers from the capitalist. As the workers are impatient, this may be attractive to them. In a Ramsey equilibrium, the workers *must* be relatively impatient, else they would be capitalists. We can resolve this question by estimating the worker's utility along the transition path from one steady state to another.

Provided the change is small, the following theorem shows that the workers do not desire to save anything, only the capitalist adjusts his saving. Further, the transition path to the new steady state is monotonic. Using the results of section three, we find:

THEOREM 4. *Let ϑ_1 be given. There is a $\eta > 0$ so that for all ϑ_2 with $|\vartheta_2 - \vartheta_1| < \eta$, there is a Ramsey equilibrium starting at the ϑ_1-steady state. In this equilibrium, none of the workers own any capital, and the capitalist's capital holdings converge monotonically to $K(\vartheta_2)$.*

Similar results could be applied to other specifications of the tax function τ, such as the tax on interest income considered earlier.

Now suppose taxes are raised from ϑ_1 to ϑ_2. Raising taxes lowers the steady-state capital stock. Thus capital is monotonically decreasing along the transition path to the new steady state. As a function of capital, consumption of household $h > 1$ is given by $c^h(K) = [\ell_h f(K) + (\theta_h\vartheta - \ell_h)f'(K)K]$. This has derivative $\theta_h\vartheta f' + (\theta_h\vartheta - \ell_h)f''K = (\theta_h K f'')[\vartheta(\varepsilon + 1) - \xi_h]$. Thus $\partial c^h(K)/\partial K$ has the same sign as $\xi_h - \vartheta(\varepsilon+1) = \xi_h - \vartheta(f'K)'/Kf''$. Since gross capital income $Kf'(K)$ is increasing in K, $\partial c^h(K)/\partial K > 0$. Consumption by the workers monotonically decreases to the steady-state value.

With consumption decreasing to the steady-state value, the workers do at least as well as they would if they received steady-state consumption from period two onward. As $\sum_{t=1}^{\infty} \delta_h^{t-1} u^h(c_t^h) \geq u^h(c_1^h) + \sum_{t=2}^{\infty} \delta_h^{t-1} u^h(c^h(\vartheta))$, household h has utility of at least $u^h(c_1^h) + \delta_h(1-\delta_h)^{-1}u^h(c^h(\vartheta_2))$ on the transition path. Under the old tax regime, the household obtained utility $(1-\delta_h)^{-1}u^h(c^h(\vartheta_1)) = u^h(c^h(\vartheta_1)) + \delta_h(1-\delta_h)^{-1}u^h(c_h(\vartheta_1))$. Subtracting shows that the gain is at least $[u^h(c_1^1) - u^h(c^h(\vartheta_1))] - \delta_h(1-\delta_h)^{-1}[u^h(c^h(\vartheta_1)) - u^h(c^h(\vartheta_2))]$.

Take a Taylor expansion about the old steady state. The approximate gain, provided the change is small enough, is proportional to $u'(c^h(\vartheta_1))\{(1-\delta_h)[c_1^h - c^h(\vartheta_1)] + \delta_h[c^h(\vartheta_2) -$

$c^h(\vartheta_1)]\}$. It is enough that $(1-\delta_h)[c_1^h - c^h(\vartheta_1)] + \delta_h[c^h(\vartheta_2) - c^h(\vartheta_1)]$ be positive. As the initial capital stock is $K(\vartheta_1)$, wages at time 1 are unchanged, and $c_1^h - c^h(\vartheta_1) = \theta_h(\vartheta_2 - \vartheta_1)f'(K(\vartheta_1))K(\vartheta_1)$. The steady-state calculations show that $c^h(\vartheta_2) - c^h(\vartheta_1)$ is approximately $\theta_h(\vartheta_2 - \vartheta_1)K(\vartheta_1)f'(K(\vartheta_1))(1+\vartheta_1\varepsilon - \xi_h)/(1-\vartheta_1)$. The approximate gain is then proportional to $(\vartheta_2 - \vartheta_1)\{(1-\delta_h)(1-\vartheta_1) + \delta_h(1+\vartheta_1\varepsilon - \xi_h)\}$ which reduces to $(\vartheta_2 - \vartheta_1)(1-\delta_h\xi_h)$ for ϑ_1 near zero. All the workers with $\xi_h\delta_h < 1$ will gain if a capital tax near zero is increased. When transfer and labor shares are both equal ($\theta_h = \ell_h = 1/H$), $\xi_h = 1$ and all of the workers gain.

Note that these calculations only provide a lower bound on the gain. A more precise description of the transition path is required if to consider large tax rates. In one case, a complete description is possible.

5. Example: Logarithmic Utility, Cobb-Douglas Technology

Now suppose the production function is Cobb-Douglas with $f(K) = K^\rho$ and utility is logarithmic, $u_h(c) = \log c$. The maintained assumptions are satisfied, and gross capital income is $Kf'(K) = \rho K^\rho$, which is increasing. Under these conditions, the symmetries of Boyd (1990) can be used to calculate the adjustment path from one steady state to another. This yields explicit formulae for the time paths of wages, rental rates, individual and aggregate capital stocks, and consumption. Given this information, the utility of each individual may be explicitly calculated for the transition path.

As can be easily verified, the following (symmetry) mapping transforms Ramsey equilibria with initial capital x^h into Ramsey equilibria with initial stock λx^h.

$$S(q_t, w_t, K_{t-1}, c_t^h, x_{t-1}^h, \vartheta, T_t, x^h) = (\lambda_t\lambda_{t-1}^{-1}q_t, \lambda_t w_t, \lambda_{t-1}K_{t-1}, \lambda_t c_t^h, \lambda_{t-1}x_{t-1}^h, \vartheta, \lambda_t T_t, \lambda x^h),$$

where $\lambda_t = \lambda^{\rho^t}$.

As above, the workers have no capital in the steady state. All the capital is owned by the capitalist. As the steady state is given by $q(1-\vartheta) = 1/\delta_1$, the capitalist owns $K(\vartheta) = [\rho\delta_1(1-\vartheta)]^{1/(1-\rho)}$ units of capital. Total taxes are $T = \rho\vartheta K^\rho$, the wage is $w = (1-\rho)K^\rho/L_0$, and worker h consumes $c^h = L_h w + \theta_h T = [\ell_h(1-\rho) + \theta_h\rho\vartheta] \times [\rho\delta_1(1-\vartheta)]^{\rho/(1-\rho)}$. A simple calculation now reveals that an increase in the property tax reduces steady-state consumption by the workers, in spite of the increased transfers. This contrasts with the after-tax rate of return to the capitalist which remains fixed.

We start in the long-run steady state with tax rate ϑ_1. The tax rate is suddenly and unexpectedly changed to ϑ_2. The new steady state has capital stock $K(\vartheta_2)$, but we start with the capitalist holding $K(\vartheta_1)$. By setting $\lambda = K(\vartheta_1)/K(\vartheta_2)$, the symmetry S will generate the transition path.

As $\lambda = [(1-\vartheta_1)/(1-\vartheta_2)]^{1/(1-\rho)}$, $K_t = [(1-\vartheta_1)/(1-\vartheta_2)]^{\rho t/(1-\rho)}K(\vartheta_2)$. Similar results hold for the behavior of the other variables. Utility is of particular interest. For household h,

$$\begin{aligned} U^h &= [\rho/(1-\rho\delta_h)]\log\lambda + U^h(c^h(\vartheta_2)) \\ &= [\rho/(1-\rho\delta_h)(1-\rho)]\log[(1-\vartheta_1)/(1-\vartheta_2)] + U^h(c^h(\vartheta_1)) \end{aligned}$$

where $U^h(c^h(\vartheta_2)) = (1-\delta_h)^{-1}\log c^h(\vartheta_2)$ is the utility in the steady state. The change in utility (compared with remaining at the old level of taxation) is

$$\Delta U = \frac{\rho}{(1-\rho\delta_h)(1-\rho)}\log\left(\frac{1-\vartheta_1}{1-\vartheta_2}\right) + \frac{1}{1-\delta_h}\log\left(\frac{c^h(\vartheta_2)}{c^h(\vartheta_1)}\right).$$

Now,

$$\frac{c^h(\vartheta_2)}{c^h(\vartheta_1)} = \left(\frac{\ell_h(1-\rho)+\theta_h\vartheta_2\rho}{\ell_h(1-\rho)+\theta_h\vartheta_1\rho}\right)\left(\frac{1-\vartheta_1}{1-\vartheta_2}\right).$$

Substituting and simplifying reveals:

$$\Delta U^h = \frac{1}{1-\delta_h}\left[\frac{\rho\delta_h}{1-\rho\delta_h}\log\left(\frac{1-\vartheta_2}{1-\vartheta_1}\right) + \log\left(\frac{\ell_h(1-\rho)+\theta_h\vartheta_2\rho}{\ell_h(1-\rho)+\theta_h\vartheta_1\rho}\right)\right]$$

Differentiating with respect to ϑ_2 and simplifying shows

$$\partial(\Delta U^h)/\partial\vartheta_2 = \rho(1-\rho\delta_h - \xi_h\delta_h(1-\rho) - \vartheta_2)/(1-\delta_h)(1-\rho\delta_h)(1-\vartheta_2)(\xi_h(1-\rho)+\vartheta_2\rho).$$

Denote the utility maximizing tax rate for household h by ϑ_h^*. For $h = 2, \dots, H$ we have $\vartheta_h^* = 1 - \delta_h + \delta_h(1-\rho)(1-\xi_h) > 0$. If $\vartheta_1 > \vartheta_h^*$, household h will gain from a tax cut to ϑ_h^*, while if $\vartheta_1 < \vartheta_h^*$, household h will gain from a tax increase to ϑ_h^*. When $\xi_h = 1$, ϑ_h^* has the simpler form $\vartheta_h^* = 1 - \delta_h$ for $h = 2, \dots, H$. The household's preferred tax rates have the same ranking as their time preference rates.

5.1. Politico-Economic Equilibrium

A closer examination of ΔU^h, $h = 1, \dots, H$, shows that ΔU^h is increasing for $\vartheta_2 < \vartheta_h^*$ and decreasing for $\vartheta_2 > \vartheta_h^*$. Preferences over tax rates are single-peaked. *All* of the workers prefer increasing taxes to any ϑ with $0 < \vartheta < \min_{h=2,\dots H} \vartheta_h^*$ to the no-tax steady state. A similar analysis of ΔU^1 shows that $t_1^* = 0$. Only the capitalist prefers no tax, and only the capitalist stands to lose from a small tax on capital. The capitalist's preferences are also single-peaked. We define a *Ramsey Politico-Economic Equilibrium* as a Ramsey Equilibrium $\{q_t, w_t, K_{t-1}, c_t^h, x_{t-1}^h, \vartheta, T_t, x^h\}_{t=1}^{\infty}$ where ϑ is preferred by a majority of the voters to any alternative tax rate $0 < \vartheta' < 1$. Since preferences are single-peaked, the median voter model applies, and a steady state Ramsey politico-economic equilibrium occurs when the tax rate ϑ^* is the median of the $\vartheta_h^* = 1 - \delta_h + \delta_h(1-\rho)(1-\xi_h)$. At that steady state, a Ramsey equilibrium obtains, and no alternative tax rate can win a majority rule election against ϑ_h^*.

6. Conclusion

This paper studies Ramsey equilibrium with fixed capital tax rates. Ramsey equilibria could easily be defined under other tax regimes. Doing this would allow consideration of progressive taxation. Sarte (1997) examines Ramsey equilibria arising under the equal sacrifice tax functions estimated by Gouveia and Strauss (1994).

References

Becker, Robert A. (1980), On the long-run steady state in a simple dynamic model of equilibrium with heterogeneous households, *Quart. J. Econ.* **47**, 375–382.

Becker, Robert A. (1985), Capital income taxation and perfect foresight, *J. Pub. Econ.* **26**, 147–167.

Becker, Robert A. and Ciprian Foias (1987), A characterization of Ramsey equilibrium, *J. Econ. Theory* **41**, 173–184.

Becker, Robert A. and Ciprian Foias (1990), Convergent Ramsey equilibria, *Libertas Mathematica* **10**, 41–52.

Becker, Robert A. and Ciprian Foias, The local bifurcation of Ramsey equilibrium, *Econ. Theory* **4**, 719–744.

Becker, Robert A., John H. Boyd III and Ciprian Foias (1991), The existence of Ramsey equilibrium, *Econometrica* **59**, 441–460.

Boadway, Robin (1979), Long-run tax incidence: A comparative dynamic approach, *Rev. Econ. Studies* **46**, 505–511.

Boyd, John H., III (1990), Symmetries, dynamic equilibria and the value function, *in* "Conservation Laws and Symmetry: Applications to Economics and Finance" (R. Sato and R. Ramachandran, eds.), Kluwer, Boston.

Chamley, Christophe (1981), The welfare cost of capital income taxation in a growing economy, *J. Polit. Econ.* **89**, 468–496.

Feldstein, Martin (1974a), Incidence of a capital income tax in a growing economy with variable savings rates, *Rev. Econ. Studies* **41**, 505–513.

Feldstein, Martin (1974b), Tax incidence in a growing economy with variable factor supply, *Quart. J. Econ.* **88**, 551–573.

Gouveia, Miguel and Robert P. Strauss, "Effective Federal Individual Income Tax Functions," Working Paper, 1994.

Grieson, Ronald E. (1975), The incidence of profits taxes in a neo-classical growth model, *J. Pub. Econ.* **4**, 75–85.

Homma, Masaaki (1981), A dynamic analysis of the differential incidence of capital and labour taxes in a two-class economy, *J. Pub. Econ.* **15**, 363–378.

Judd, Kenneth L. (1985), Redistributive taxation in a simple perfect foresight model, *J. Pub. Econ.* **28**, 59–83.

Pasinetti, Luigi L. (1962), Rate of profit and income distribution in relation to the rate of economic growth, *Rev. Econ. Studies* **29**, 267–279.

Sarte, Pierre-Daniel G. (1997), Progressive taxation and income inequality in dynamic competitive equilibrium, *J. Pub. Econ.* **66**, 145–171.

Sorger, Gerhard (1994), On the structure of Ramsey equilibrium: Cycles, indeterminacy, and sunspots, *Econ. Theory* **4**, 745–764.

Wan, Henry Y., "Economic Growth," Harcourt, Brace and Jovanovich, New York, 1971.

Wildasin, David, "On the Analysis of Labor and Capital Income Taxation in a Growing Economy with Government Saving," Working Paper, Indiana University, 1984.